

(An ISO 3297: 2007 Certified Organization)

Website: <u>www.ijirset.com</u>

Vol. 6, Issue 7, July 2017

Wing Beat Frequency of Avian Fliers

Nasiha Saher Bano¹, Ahmed Waheedullah², Adeel Ahmad³

Research Scholar, Rayalaseema University, Kurnool, India¹

Professor of Mathematics, Department of Science & Humanities, Lords Institute of Engineering & Technology,

Hyderabad, India²

Professor of Physics (Retd), Biophysics Research Laboratory, Department of Physics, Nizam College (Autonomous),

Osmania University, Hyderabad, India.³

ABSTRACT: The paper presents the comparative study on wing beat frequency of species of birds. Frequency of wing beat of avian fliers is calculated using different theories and the data is compared and discussed. The paper suggests that Mass flow theory is superior to any other theory proposed for the computation of wing beat frequency.

KEYWORD: Wing beat frequency, Mass flow theory, Greenewalt's theory, Deakin's theory, Crawford's theory, Birds.

I. INTRODUCTION

Wing beat frequency of a bird is an important parameter from the point of view of aerodynamics and bioenergetic. Several attempts have been made to give the quantitative expressions for the relation between body and wing beat frequency of insects, birds and bats. Morphology, sex, age, temperature, pressure, and fatigue are some of the factors that affect the frequency of the insect flight sound [1]. Wing beat frequency is a species specific parameter [2,3]. Reed et. al., [4] and Sotavalta [5] proposed empirical relations for the wing beat frequency of insects in terms of their physical parameters such as mass, length and stroke angle. Sotavalta [6] reported that high wing beat frequency of *Lipidoptera* and *Odonata* has inverse relation with the moment of inertia of the wing. But the change was much distinct in *Diptera* and *Hymenoptera*. In *Periplaneta americana* the wing beat frequency is unaffected by moment of inertia of the wings. The aim of the present study is to evaluate the applicability of proposed theories, mostly developed for insects, for the computation of wing beat frequency of small and large size birds of Indian origin.

II. THEORIES OF WING BEAT

2.1. Mechanical oscillator theory

A flier, whether it may be insect, a bird or a bat can be considered as mechanical harmonic oscillator to compute its wing beat frequency.

A differential equation can be set up for damped oscillations as

$$I\frac{d\phi}{dt^2} + B\frac{d\phi}{dt} + f\phi = F_0 \cos \omega t \tag{1}$$

I = sum of internal and external moments of inertia

 \emptyset = wing stroke angle

B = damping parameter

t = time

f = restoring force

 $F_0 = driving force$

 ω = angular frequency



(4)

(7)

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

If the oscillations of the wings of the flier are free and undamped, then B=0 and $F_0=0$. The differential equation 5 reduces to

$$I\frac{d^2\phi}{dt^2} + f\phi = 0 \tag{2}$$

The solution for the undamped oscillations is given by

$$\omega^2 = K \frac{b_0 r^2}{I} \tag{3}$$

where ω = angular frequency; b_0 = length of unstrained muscle; r = effective radius of attack; I = sum of internal and external moments of inertia.

Using Sotavalta's [6] data on insects, Magnan's [7] data on birds, Greenewalt [8] reported the dependence of the wing beat rates on the size of the dimensionally similar groups of flier in consistent with equation (3). Assuming $b_0 \propto l$, $r \propto 1$ and $m \propto 1^3$, he proved that $I \propto 1^5$ so that the equation (3) reduces to

vl = constant.

where
$$v = \frac{\omega}{2\pi}$$

According to Greenewalt, the relation (4) holds good for insects alone, while for humming birds the anatomical relationship appears to be quite different. According to him, for birds, the mass (m) of the flier is proportional to $l^{1.5}$ and not l³. Taking $r \propto l$ and $b_0 \propto m^{1/2}$ equation (4) can be written as

v. $l^{1.25} = constant$ (5) Further, he generalizes the frequency - wing length expression for insects and birds as (6)

 $v l^n = constant$

where n lies between 1 and 1.25. Probably on the basis of graphical relation between the anatomical parameters of fliers, he puts down the relation as

 $v l^{1.15} = 3540$

where 'l' is the wing length expressed in millimeters.

2. 2. Wing beat frequency on the basis of dimensional analysis

An expression is derived [9] for the wing beat frequency by considering the basic dimensional analysis. Deakin worked on several species of fliers having typical mass, wing area and wing beat frequencies under varying flight conditions. For theoretical computation of wing beat frequency, he considered those species which are geometrically similar,

having the same mass distribution. He considered two dimensionless ratios $\frac{\rho A^{3/2}}{m}$ and $\frac{v^2 A^{1/2}}{g}$ and using

'Buckingham Pi theorem,' he obtained the following equation

$$F(\rho A^{3/2} / m, v^2 A^{1/2} / g) = 0$$
(8)
$$v^2 A^{1/2} / g = \phi(g A^{3/2} / m)$$
(9)

or

$$v^{2}A^{1/2} / g = \phi(\rho A^{3/2} / m)$$
(9)
$$v = g^{1/2}A^{-1/4} \chi(\rho A^{3/2} / m)$$
(10)

Therefore $v = g^{1/2} A^{-1/4} \chi(\rho A^{3/2} / m)$ for some function χ of the given argument, where

- ρ = density of the air, (ML⁻³)
- m = mass of the species, (M)
- g = acceleration due to gravity, (LT⁻²)

v = frequency of the wing beat, (T⁻¹)

and A = effective area of the wings, (L²)



(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

The dimensionless ratio $\rho A^{3/2}/m$ is the ratio of the mass of a portion of air (roughly the same size of flier) to the mass of the fliers itself from Frobenius series for (13), we have

$$\chi(x) = Kx^{\alpha}(1 + a_1 x + a_2 x^2 + \dots)$$
(11)

using approximations

$$\chi(x) = K x^{\alpha} \tag{12}$$

and
$$x = \rho A^{3/2} / m$$
 (13)

and substituting in equation (14), we get

$$v = g^{1/2} A^{-1/4} K \rho^{\alpha} A^{3\alpha/2} m^{-\alpha}$$
⁽¹⁴⁾

Considering L = m/A as the flier load per unit of wing area, equation (14) reduces to

$$v = g^{1/2} \rho^{\alpha} K m^{(\alpha/2 - 1/4)} L^{\left(\frac{-3\alpha}{2} + \frac{1}{4}\right)}$$
(15)

Therefore, $\nu \propto L^{(-3\alpha/2+1/4)}$ (16)

On the basis of experimental data he found that

$$v \propto L$$
 (17)
From (19) and (20), we have

$$\frac{-3\alpha}{2} + \frac{1}{4} = 1$$
(18)

Therefore
$$\alpha = \frac{-1}{2}$$
 (19)

Substituting the value of α in equation (18)

$$v = g^{1/2} \rho^{-1/2} K m^{1/2} A^{-1}$$

$$v = K (g / \rho)^{1/2} \frac{m^{1/2}}{A}$$

$$v = k m^{1/2} / A$$
(20)

where
$$k = K \left(\frac{g}{\rho}\right)^{1/2}$$
 (21)

Deakin compared equation (20) with that of Roeder's [10] equation, under his assumptions and found that

$$v = \frac{km^{1/2}}{A^{5/4}}$$
(22)

In view of the approximate nature of many assumptions, this agreement between the two methods may be well regarded. From equation (18), it is inferred that $v \propto \rho^{-1/2}$ for the same insect. The equation (15) is considered to be reliable. Referring the Rashevsky's [11] data, Deakin found that 'K' has a value of 65 in C.G.S units.

Using equation (22) to honey bee (Apis sp.) with $m \cong 0.001$ gm, $A \cong 0.006$ cm², he found $v \cong 350$ Hz., whereas the experimental value was 200-250 Hz. This is in fact an over estimation. In order to seek reasons for this over estimation, Deakin discussed the muscle nerve interaction when an insect is air borne. According to his investigations, the maximum rate of muscle response in bee when it is air borne is about 35 Hz which is well short of what is required in the case of honey bee. According to him, when the frequency is in the range of 400 Hz, we entered a new region of impossibility-no nerve can deliver the impulses fast enough. Such "impossible" frequencies are required for the bumble-bee (Bombus) to enable flight. Deakin states "it is mathematical impossible for a bumble bee to fly". He pointed out that the error in frequency is due to the physiological condition of the flier. This can not consist in the muscles really being able to respond faster to nervous impulses. This is in fact not observed. It is found that a few



(An ISO 3297: 2007 Certified Organization)

Website: <u>www.ijirset.com</u>

Vol. 6, Issue 7, July 2017

groups of insects possess an atypical type of flight muscles, termed as asynchronous flight muscles, as their contraction is not in response to the nervous stimulus but a direct response to the load on the wing. The above assumption has been proved by the mutilation of the wing. As the wing area is decreased there is an increase in the frequency. If frequency were dependent on a neural impulse, this result might not be expected, as the rate of such impulses is internally rather than externally controlled.

Pringle [12] provided conclusive demonstration by monitoring the nervous impulses to the wing and found that the rate of nervous impulses was unequal to the wing beat frequency. The rate at which the nerve impulses arrive was, in fact, similar to the rate at which they are delivered to normal (synchronous) flight muscles.

The existence of a physiological distinct type of muscle was recognized with this work. In fact, it might have been predicted. The key to a discovery of major importance in the study of muscle physiology turns out from the old saying of the obviously incorrect assertion "Bumble bees can not fly".

2. 3. A simple model on wing beat using Newton's Law

Crawford [13] set up a relation to compute the wing beat frequency by using Newton's laws.

Consider a hovering insect, the weight of which is supported by the reaction of the wings against the flow of air pushed down by the wings.

$$mg = \frac{d}{dt}(M_{air}V_{air}) \tag{23}$$

where $M_{air} = mass$ of air; $V_{air} = velocity$ of air; mg = weight of insect

Let the wing stroke be made as effective as possible. Assuming that up stroke is not effective as it is only a recovery stroke. But on the down stroke mass of the air pushed down is ' ρ ' times the volume swept out by the wing, where ' ρ ' is the density of air. This volume is the wing area (A) times the amplitude z_0 of the wing stroke.

Thus, $M_{air} = 2\rho A z_0$

Assume that this air, pushed down, attains the maximum velocity ωZ_0 . This takes place one in each period so that 2π

$$dt = -\frac{1}{\omega}$$

or

If we average over one cycle, equation (23) can be written as

 $mg = (2\rho A Z_0)(\omega Z_0)(\omega / 2\pi)$ $mg = \frac{\rho A Z_0^2 \omega}{\pi}$ (24)

For a reasonable wing shape, taking the amplitude as large as possible and $Z_0 = \sqrt{A}$, equation (24) becomes

$$\omega^2 = \frac{\pi mg}{\rho A^2} \tag{25}$$

Equation (25) is similar to Deakin's equation for the wing beat except for Deakin's equation being the result of dimensional analysis has an unknown dimensionless numerical factor 'K' of order unity which he evaluates by considering the data presented by Rashevsky. In this relation there are no undetermined constants.

2.4. Mass flow theory

Puranik, et. al., [14] developed a theory for the evaluation of the wing beat frequency of a flier in its normal state of hovering on the basis of mass flow of air induced downwards due to wing beat.

When the flier is in the state of hovering, the system is said to be in the state of 'dynamical equilibrium' which is achieved by the flier by generating a 'reacting force', R (i.e., generated to the wing beat), to just overcome its own weight. The system in bio-aerodynamics comprises of the flier and the induced air, which constitutes action-reaction pair, for the hovering flight. The reacting force R, so generated due to wing beat is proportional to the following:

1. The disc area, S_d of the flier



(27)

(28)

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

where
$$S_d = \pi \left(\frac{\text{Wing span}}{2}\right)^2$$

- 2. The effective wing breadth, B_{eff}
- 3. The density, ρ of the medium in which it is hovering
- 4. The half of wing beat frequency, $v_h/2$, (the factor half enters in because down stroke alone is effective in inducing the air downwards).

Therefore, the reacting force R is proportional to the rate of mass flow of air induced downwards, i.e.,

$$R \propto S_d \cdot B_{eff} \cdot \rho \cdot v_h / 2 \tag{26}$$

or
$$R = K.S_d.B_{eff}.\rho v_h / 2$$

where 'K' is the proportionality constant, the dimensions being LT^{-1} . For the hovering state, R = M_f, g

where 'M_f' is the mass of the flier and 'g' is acceleration due to gravity. From equations (27) and (28), it follows that $M_{f} \cdot g = K \cdot S_{d} \cdot B_{eff} \cdot \rho \cdot v_{h} / 2$

Or
$$V_h = \frac{2g}{K} \frac{M_f}{S_d B_{eff} \cdot \rho}$$
(29)

By putting $S_d = \pi L^2/4$ where l is the wing span of the flier in equation (30) We have $M = \alpha = (K/8) \pi L^2 R_{--} \alpha u$

$$M_{f} \cdot g = (K / 8) \cdot \pi \cdot L^{*} \cdot B_{eff} \cdot \rho \cdot v_{h}$$

Therefore, $v_{h} = \left(\frac{8g}{K\pi\rho}\right) \left(\frac{M_{f}}{L^{2} \cdot B_{eff}}\right)$
 $v_{h} = \frac{K' M_{f}}{L^{2} \cdot B_{eff}}$, where $K' = \frac{8g}{K\pi\rho}$ (30)

The value of K is determined experimentally by drawing a curve between M_f and v₀.L².B_{eff} (all in C.G.S units) for insects, where v₀ is observed the wing beat frequency of the flier in its hovering state and the slope of which gives the value of K as 2086.

Thus the relation (30) gives the hovering frequency in terms of the mass of the flier, the wing span, the effective wing breadth and the density of the air. This is different from Greenewalt's relation which gives the frequency in terms of the wing length alone. Hence the relation for the wing beat frequency in the normal hovering state is

$$v_h = \frac{(2086)M_f}{L^2.B_{eff}}$$
(31)

For computing the value of the constant K', insects are chosen due to the fact that their frequencies can be measured accurately by the available experimental techniques. But in the case of other fliers which have low values of hovering frequencies (birds and bats) it is rather difficult, if not impossible, to determine their hovering frequencies under laboratory conditions. Particularly in such cases equation (31) is to be very useful. In order to verify equation (31), Adeel Ahmad [15] measured the body parameters such as M_{fr} . L and B_{eff} of five species of insects namely <u>Periplaneta Americana</u>, <u>Tesseratoma javanica</u>, <u>Apis</u> sp., <u>Chrysocoris purpureus</u> and <u>Bombus</u> sp. The wing beat frequency of the above fliers have also been determined by recording their flight sound generated in the tethered state.

A curve is drawn between the mass of the flier M_f and $v_0 L^2 B_{eff}$ which is found to be linear and passes through the origin indicating the validity of the theory. The slope of which gives the value of K 'which can be further used for computing the wing beat frequency of the fliers such as birds and bats whose wing beat frequency in hovering state is rather difficult to determine under laboratory conditions. For large insects, birds and bats, the wing stroke angle is considerably large (120°-140°), the wing swept area 'S_w' is approximately equal to the disc area 'S_d'. But it cannot be true to those insects where the wing stroke angle is small (about

60°). Hence in the mass flow theory, $S_w = \text{Stroke}$ angle x (Wing length)², is considered instead of disc area, $S_d = \frac{\pi L^2}{\Lambda}$. Then,



(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

equation (30) for the wing beat frequency of the high frequency fliers (such as mosquitoes) as in accordance to mass flow theory is,

$$v_h = \frac{2M_f \cdot g}{K \cdot S_w \cdot B_{eff} \cdot \rho}$$

(32)

III. MATERIALS AND METHODS

The large, medium and small size birds of Indian origin were collected in and around Hyderabad. Length (l) and span (tip to tip distance, L) were measured using measuring tape. Area of the wing was measured taking its contour of the wing on white paper and using planimeter. Mass of the flier is determined using electronic weighing machine. Based on the observation on wing kinematics, the stroke angle was approximated to 120^{0} irrespective of size of the flier. Wing beat frequency of small size birds was evaluated using electronic stroboscope, while for some of the medium and large size birds noted from available data in literature [16].

IV. RESULTS AND DISCUSSION

Table 1 presents data on body parameters such as body mass; length, span, breadth and area of the wing of 25 species of large, medium and small size birds of Indian origin. This basic data is used for the calculation of wing beat frequency of different theories. The body mass of the birds of the study varies from 6 - 306 gm. The wing length is in the range of 7 to 42 cm. The wing span is approximately double, if body breadth is neglected. The values of effective wing breadth lie in the range of 3 to 17 cm. The wing area ranges from 40 to 1311 cm².

Flier	$M_{\rm f}$	L	B _{eff}	1	Α
	(gm)	(cm)	(cm)	(cm)	(cm^2)
Corvus splendence	306	78	17	39	1311
Psittacula krameri	106	52	9	26	446
Columba livia	258	63	12	31	741
Falco sp.	135	56	10	28	567
Grus sp.	149	50	11	25	567
Bubo sp.	124	53	10	26	517
Passer domesticus	22	22	5	11	118
Loncura malacca	16	18	3	9	54
Estrilda amandava	9	16	3	8	40
Loncura malabarica	13	17	3	8	47
Loncure punculata	14	18	3	9	51
Egeretta garzetta	272	81	13	38	927
Bubulaus ibis	292	91	14	42	1143
Ardeola gravii	276	76	13	35	904
Acridotherus tristis	95	46	9	21	372
Psittacula cathorpae	49	50	6	19	207
Peridicula astatica	38	29	5	13	131
Dicrurus adsimilis	37	43	9	19	329
Turdoides striatus	51	32	7	13	184
Nactorinia asiatica	6	16	5	7	49
Harops orientalis	17	29	6	13	161
Coracias indica	103	59	13	28	551
Upupa apops	39	39	10	18	323
Apus affinis	18	30	5	14	105
Malasittacus undulatus	29	28	5	12	89

Table 1 - Basic Body Parameters



(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

Table 2 compares the wing beat frequencies of 25 species of birds, calculated using theories proposed by Greenewalt, Deakin, Crawford and Puranik et. al. For the verification of calculated values of wing beat frequency, in the case small size birds, it is measured experimentally under stroboscopic flash. The observed data of some birds is taken from literature. The data on wing beat frequency is presented in Table 2.

		1	-	0	
Flier	ν_h	$\nu_{\rm D}$	$\nu_{\rm C}$	ν_{m}	ν_{o}
	(Hz)	(Hz)	(Hz)	(Hz)	(Hz)
Corvus splendence	3	132	128	5	5
Psittacula krameri	6	300	129	10	10
Columba livia	4	248	156	11	11
Falco sp.	6	250	129	9	8
Grus sp.	6	263	136	11	9
Bubo sp.	5	269	130	10	10
Passer domesticus	15	720	113	18	17
Loncura malacca	20	1632	143	34	30
Estrilda amandava	24	1781	576	35	40
Loncura malabarica	21	1750	142	42	45
Loncure punculata	20	1640	140	41	40
Egeretta garzetta	4	192	144	8	6
Bubulaus ibis	3	153	134	6	6
Ardeola gravii	4	200	147	9	10
Acridotherus tristis	7	356	134	13	10
Psittacula cathorpae	8	532	129	9	8
Peridicula astatica	12	831	143	24	20
Dicrurus adsimilis	9	259	89	6	5
Turdoides striatus	13	630	139	18	15
Nactorinia asiatica	25	1129	94	13	10
Harops orientalis	13	430	86	9	8
Coracias indica	6	227	115	6	5
Upupa apops	9	272	92	6	6
Apus affinis	12	754	110	11	12
Malasittacus undulatus	15	1177	151	20	20

Table 2 – A comparison on frequency of wing beat

Frequency of wing beat of a flier is a basic requirement for the flight in hovering state, without which the hovering flight of a flier cannot be imagined. There exist, in literature, several theories for the determination of frequencies of wing vibration of natural fliers. Every theory has its own merits and draw backs.

As is known, wing beat frequency is a function of the body mass, wing span and wing breadth. Hence any variation in these parameters should introduce corresponding variations in the wing beat frequency of the flier. But mechanical oscillation theory, proposed by Greenewalt, assumes that wing beat frequency is function of wing length alone. The perfect mechanical oscillating system, the frequency of oscillation should be proportional to inertia and should slightly be affected by changes in the damping. Hence it must be emphasized that the high wing beat frequency of insects is not determined by any one physical factor. There exists a balance between physical and physiological characteristics under the conditions of flight. If this balance is upset experimentally, by the changes in wing area, air density, or factors which affect the physical or physiological properties of the tissues, the frequency may change and thus so particularly in insects with asynchronous flight muscles where the rhythm is directly affected by the physical characteristics of the effective load.

To verify the semi-empirical relation thus developed, Greenewalt has confined his experiments to only one flier namely female ruby throat. Greenewalt's theoretical approach becomes insignificant when he starts assuming different



(An ISO 3297: 2007 Certified Organization)

Website: <u>www.ijirset.com</u>

Vol. 6, Issue 7, July 2017

proportionalities for b_0 , r, m and I because of non-singular surface of the flier, both from the point of view of density and regularity of contour. Besides, the frequency of a flier, in general, appears to depend upon the state of flight more so in the case of humming birds, the wing beat frequency ranges from 55 Hz to 200 Hz depending upon the state of flight [15] and as such cannot be considered as a constant under different flight conditions. Therefore, the flight appears to be associated not with a single frequency but with a frequency spectrum.

The expression derived by Deakin, [9] for wing beat frequency, by considering the basic dimensional analysis, cannot be considered as universal for several insects, birds and bats due to the fact that for the computation of wing beat frequency Deakin considered those species which are geometrically similar. The relation set up by Crawford [13] to compute wing beat frequency, by using Newton's law, is similar to Deakin's equation. In this relation there are no undetermined constants. The wing beat frequency calculated using Deakin's and Crawford's theories shows very large deviation from the observed values. Hence, these two theories can not be used for the calculation of wing beat frequency.

V. CONCLUSIONS

- 1. Mechanical oscillation theory, proposed by Greenewalt, assumes that wing beat frequency is function of wing length alone. It is not true for a variety of natural fliers (insects, birds and bats).
- 2. Theory based on dimensional analysis can be applicable to geometrical similar fliers, not all.
- 3. Theory based on Newton's laws shows a large deviation from the observed data and hence can not be applied to natural fliers.
- 4. The wing beat frequency calculated for insect, bird and a bat using Mass flow theory is in good agreement with experimental values. It can be applied to natural fliers, starting from smallest size flier *mosquito* to largest size flier *Falcon* or *wild crow*.
- 5. Mass flow theory explains the phenomenon of hovering flight of insects, birds and bats more adequately than any other theory reported in literature.
- 6. Further, it is to be concluded that the wing beat frequency of a flier, in general, varies with the state of flight.
- 7. The most striking feature of mass flow theory is that explains the effect of body phenomenon and environmental influences on the wing beat frequency of fliers when they are in the state of hovering.

REFERENCES

[1] Adeel Ahmad, "Effect of body phenomenon and environmental influences on flight sound of the bug *Tesseratoma javanica* Thunberg". Ind. J. Exp. Biol., Vol. 20, No. 9, pp. 682 - 686,1982.

[2] Adeel Ahmad, "A comparative study on flight surface and aerodynamic parameters of insects, birds and bats", Ind. J. Exp. Biol., Vol. 22, pp. 270–278, 1982.

[3] Syed Najmul Hassan, N. Chari and Adeel Ahmad, "Analysis of wing vibration of mosquito *Anopheles stephensi*", Entomon, Vol. 8, No. 1, pp. 47, 1983.

[4] S. C. Reed, C. M. Williams, L. E. Chadwick, "Frequency of wing-beat as a character for separating species races and geographic varieties of *Drosophila*", Genetics, Vol. 27, pp. 349-361, 1942.

[5] O. Sotavalta, "Effect of temperature on flight speed of the blowfly", Acta. Ent. Fenn., Vol. 4, pp.1,1947.

[6] O. Sotavalta, Acoustic behavior of animals (ed. R.G. Busnel), Elsevier Publishing Co., New York, 1963.

[7] A. Magnan , Le Vol des. Insects, Paris, pp.186, 1934.

[8] C. H. Greenewalt, "The wings of insects and birds as mechanical oscillators", Proc. Amer. Phil. Soc., Vol. 104, pp. 605, 1960.

[9] M. A. B. Deakin, Amer. J. Phy., Vol. 38, pp.1003,1970.

[10] Roeder, "Movements of the thorax and potential changes in the thoracic muscles of insects during flight", Biol. Bull. Woods. Hole, Vol. 100, pp. 95, 1953.

[11] Rashevsky, "Mathematical Biophysics", Vol. 2, Dover, New York, 1960, pp. 274.

[12] J. W. S. Pringle, "The Physiology of Insecta.", Vol. 2, (ed.) M. Rockstin, Academic Press, New York, 1965.

[13] F. S. Crawford, Am. J. Phy., Vol. 39, pp. 584, 1971.

[14] P. G. Puranik, G. Gopala Krishna, Adeel Ahmad and N. Chari, "Wing beat frequency of a flier—Mass flow theory", Proc. Ind. Acad. Sci., Vol. 85B, No. 5, pp. 327, 1977.

[15] Adeel Ahmad, "Studies on Bio-acoustics and Aerodynamics of natural fliers", Ph. D. Thesis, Osmania University, Hyderabad, India, 1978.

[16] Oliver, L. Austin, "Birds of the world", Paul Hanlyn, London, pp. 169, 1966.